The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint

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Abstract

Little attention is given in the literature to decisions on the appropriate selection of suppliers, and on assigning order quantities to these suppliers, in the case of multiple sourcing, with multiple criteria and with suppliers’ capacity constraints. Only a few mathematical programming models to analyse such decisions have been published to date, and these have tended to consider only net price as the cost of purchasing, although the costs of transportation, ordering and storage may be significantly important to the decision. In this paper a mixed integer non-linear programming model is presented to solve the multiple sourcing problem, which takes into account the total cost of logistics, including net price, storage, transportation and ordering costs. Buyer limitations on budget, quality, service, etc. can also be considered in the model. An algorithm is proposed to solve the model, and the model is illustrated using a numerical example. © 2001 Published by Elsevier Science B.V.

Keywords: Logistics; Sourcing; Criteria; Constraints

1. Introduction

In most industries the cost of raw materials and component parts constitutes the main cost of a product, such that in some cases it can account for up to 70% [1]. In high technology firms, purchased materials and services represent up to 80% of total product cost [2]. Thus the purchasing department can play a key role in an organization’s efficiency and effectiveness because it has a direct effect on cost reduction, profitability and flexibility of a company.

Selecting the right suppliers significantly reduces the purchasing cost and improves corporate competitiveness, which is why many experts believe that the supplier selection is the most important activity of a purchasing department [3,4].

In spite of the importance of supplier selection problems only a few articles have addressed the decision making. Weber and Current [5] stated that only 10 articles analysed the problem up to the time of their review. A comprehensive review of the articles which have addressed the problem can be found in Ghodsypour [6] and Ghodsypour and O’Brien [7]. The most important articles are described below.
Moore and Fearon [8] stated that price, quality and delivery are important criteria for supplier selection and they explained that linear programming can be applied to this decision making. They also discussed other applications of computer technology in the purchasing area.

Gaballa [9] is the first author who applied mathematical programming to vendor selection in a real case. He used a mixed integer programming model to formulate this decision making problem for the Australian Post Office. The objective of this programming is to minimize the total discounted price of allocated items to the vendors, under constraints of vendors' capacity and demand satisfaction.

Anthony and Bufa [10] developed a single objective linear programming model to support strategic purchasing scheduling (SPS). The linear model minimizes total cost by considering limitations of purchasing budget, vendor capacities and buyer's demand. Price and storage cost are included in the objective function. The costs of ordering, transportation and inspection are not included in the model.

Bufa and Jackson [11] presented a multi-criteria linear goal programming model for supplier selection. In this model two sets of factors are considered: (1) supplier attributes, which include quality, price, service experience, early, late and on-time deliveries and (2) the buying firm's specification, including material requirement and safety stock.

Bender et al. [12] applied single objective programming to develop a commercial computerized model for vendor selection at IBM. They used mixed integer programming, to minimize the sum of purchasing, transportation and inventory costs by considering multiple items, multiple time periods, vendors' quality, delivery and capacity. In this model quantity discount also is included. No mathematical formulations were presented and they did not indicate the kind of discount.

Narasimhan and Stoynov [13] applied a single objective, mixed integer programming model to a large manufacturing firm in the Midwest, to optimize the allocation procurement for a group of vendors. The objective of this model is to minimize the sum of the shipping and the penalty costs. The model constraints are related to vendors' production capabilities and demand.

Kingsman [14] stated that one of the most important problems which has received little attention from OR practitioners is the purchasing of materials whose prices are continually fluctuating in a stochastic manner over time. He discussed conceptually linear programming and dynamic programming as tools for purchasing raw materials with fluctuating prices.

Turner [15] presented a single objective linear programming model for British Coal. This model minimized the total discounted price by considering the vendor capacity, maximum and minimum order quantities, demand, and regional allocated bounds as constraints.

Pan [16] proposed multiple sourcing for improving the reliability of supply for critical materials, in which more than one supplier is used and the demand is split between them. Most purchasing managers agree that buying from more than one vendor will protect the buying firm in the case of shortages. Pan [16] used a single objective linear programming model to choose the best suppliers, in which three criteria are considered – price, quality and service. The total cost is taken into account as an objective function and quality and service are considered as constraints.

Sharma et al. [17] proposed a non-linear, mixed integer, goal programming model for supplier selection. They considered price, quality, delivery and service in their model, in which all criteria are considered as goals. The cost goal is decreased in relation to the increase in purchased quantity and is raised in relation to the increase in quality level.

Seshadri et al. [18] developed a probabilistic model to represent the connection between multiple sourcing and its consequences, such as number of bids, the seller's profit and the buyer's price. Only one criterion, cost, is considered in this model and the authors stated that the user should transfer the other criteria such as quality, delivery, etc., into an equivalent price.

Benton [19] developed a nonlinear program and a heuristic procedure using Lagrangian relaxation for supplier selection under conditions of multiple items, multiple suppliers, resource limitations and quantity discount. The model objective is to minimize the sum of purchasing costs, inventory...
carrying costs and ordering cost. Storage and investment limitations are considered as constraints.

Hong and Hayya [20] analysed the JIT purchasing environment. As the need for small lots is an important issue in this system, they discussed splitting a large order quantity into multiple deliveries or multiple suppliers to reduce the lot size. Their main objective was reducing the cost, hence they solved the problem by considering two important assumptions, the first is that the ordering cost of \( N \) suppliers is equal to, or less than, \( N \) times one supplier’s ordering cost. The second assumption is that the purchasing price must be less than a fixed value. As they solved the problem for the special case, it cannot be used for general situations.

Chaudhry et al. [21] developed linear and mixed integer programming for supplier selection. In their model price, delivery, quality and quantity discount are included. The objective of the model is to minimize aggregate price by considering both cumulative and incremental discounts. Quality and delivery are included as constraints.

Weber and Current [5] used multiobjective linear programming for supplier selection to systematically analyse the trade-off between conflicting factors. In this model aggregate price, quality and late delivery are considered as goals, and two sets of constraints are taken into account: (1) systems’ constraints, which are defined as the constraints which are not directly under the control of the purchasing managers such as vendor capacities, demand satisfaction, minimum order quantities established by the vendors and the total purchasing budget; and (2) policy constraints, including maximum and/or minimum order quantities purchased from a particular supplier, and the maximum and/or minimum number of vendors to be employed.

Current and Weber [22] proposed that mathematical constructs of facility location modeling can be applied to supplier selection. They did not solve any special vendor problem but they showed the similarities between the vendor selection problem and facility layout models. The complexity of both location models and supplier selection problems indicates that fitting these two methods together cannot be easy.

Rosenthal et al. [23] developed a mixed integer programming model to solve the vendor selection with bundling, in which a buyer needs to buy various items from several vendors whose capacity, quality and deliveries are limited and who offer bundled products at discounted prices. They used single objective programming and considered price, quality, delivery and suppliers’ capacity as criteria in their model.

Ghodsypour and O’Brien [7] developed a decision support system (DSS) for reducing the number of suppliers and managing the supplier’s partnership. They used integrated analytical hierarchy process (AHP) with mixed integer programming and considered suppliers’ capacity constraint and the buyers’ limitations on budget and quality etc. in their DSS.

Ghodsypour and O’Brien [24] proposed a model to deal with supplier selection, multiple sourcing, multiple criteria and discounted price. They considered the effects of limitations on budget, quality and suppliers’ capacity.

Ghodsypour and O’Brien [25] developed an integrated AHP and linear programming model to help managers consider both qualitative and quantitative factors in their purchasing activity in a systematic approach. They proposed an algorithm for sensitivity analysis to consider different scenarios in this decision making.

Most of these articles considered net price as the cost of logistics in their models, although the storage, transportation and ordering costs are also important in this decision making [26,27]. Only two articles [19,20] involved ordering and storage costs in their models. Benton [19] did not consider the supplier’s capacity and quality constraints. Hong and Hayya [20] discussed reducing lot size in the JIT environment, which is different from a general model for the supplier selection problem. These two articles considered single objective model in their work, which consider one criteria as the objective and the other criteria as the constraint in the programming. In this situations the criteria which are considered as constraints are weighted equally which rarely happens in practice.

In this present article, first a single objective model is developed to minimize the total cost of logistics, including aggregate price, ordering, and inventory costs, subject to suppliers’ capacity constraints and the buyers’ limitations on budget,
quality, delivery, etc. Second, a multiple objective programming approach is discussed to take into account different weights for various criteria.

2. Formulating the single objective model

Before describing the model the following notations are defined:

- \( D \) annual demand
- \( Q \) ordered quantity to all suppliers in each period
- \( Q_i \) ordered quantity to \( i \)th supplier in each period
- \( T \) length of each period
- \( T_i \) part of period in which the lot of \( i \)th supplier \((Q_i)\) is used
- \( r \) inventory holding cost rate
- \( X_i \) percent of \( Q \) assigned to \( i \)th supplier
- \( n \) number of suppliers
- \( A_i \) ordering cost of \( i \)th supplier
- \( P_i \) price of \( i \)th supplier
- \( C_i \) annual capacity of \( i \)th supplier
- \( q_i \) perfect rate of \( i \)th supplier
- \( q_a \) minimum accepted perfect rate of incoming parts.

In this model it is considered that the buyer would like to choose the best suppliers from among the \( n \) vendors whose capacities are limited. The objective is to minimize the sum of purchasing price, ordering costs and storage costs, subject to limitations on buyer’s budget, quality, service etc. It is also assumed that after finishing the lot of the \( i \)th supplier, the \((i + 1)\)th supplier’s lot will be received.

For simplicity assume that only one supplier is available and the buyer has to buy from this sole supplier, and its ordering cost is the sum of the ordering cost of these three suppliers, \((A = A_1 + A_2 + A_3)\). In this case the economic order quantity will be

\[
Q = \frac{2DA}{rP}.
\]

Now, \( Q \) should be split between \( n \) suppliers to minimize the total cost. The inventory level for the cases of 1 and 3 suppliers are compared in Fig. 1. In a general case, as the demand is constant and \( X_i \) is the percent of \( Q \) assigned to the \( i \)th supplier, it can be stated that

\[
Q = \sum_{i=1}^{n} Q_i,
\]

\[
Q_i = X_i Q, \quad i = 1, 2, \ldots, n,
\]

\[
T_i = X_i T, \quad i = 1, 2, \ldots, n,
\]

\[
0 \leq X_i \leq 1, \quad i = 1, 2, \ldots, n,
\]

\[
\sum_{i=1}^{n} X_i = 1.
\]

The total annual purchasing cost (TAPC), which includes ordering cost, holding cost and purchasing price, can be stated as

\[
\text{TAPC} = \text{annual ordering cost} + \text{annual holding cost} + \text{annual purchasing cost}.
\]

These costs are as follows.

2.1. Annual ordering cost (AOC)

As the product is purchased from \( n \) suppliers, the ordering cost each period (OCP) is

\[
\text{OCP} = \sum_{i=1}^{n} A_i Y_i,
\]

where

\[
Y_i = \begin{cases} 
 0 & \text{if } X_i = 0, \\
 1 & \text{if } X_i > 0, 
\end{cases}
\]

\( i = 1, 2, \ldots, n. \)

By considering that:

Annual ordering cost

\[
= \text{(ordering cost per period)} \times \text{(number of periods per year)}.
\]

Fig. 1. Comparison of inventory levels between one and three suppliers.
It can be said that AOC is

$$\text{AOC} = \text{OCP} \times \frac{1}{T},$$

$$\text{AOC} = \left( \sum_{i=1}^{n} A_i Y_i \right) \frac{1}{T} = \left( \sum_{i=1}^{n} A_i Y_i \right) D \frac{D}{Q}.$$ 

2.2. Annual holding cost (AHC)

By considering Fig. 1, it can be stated that the average inventory level for each supplier and its cost in related time are

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Time</th>
<th>Average inventory</th>
<th>Average holding cost in $T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$T_1$</td>
<td>$X_1 Q/2$</td>
<td>$(X_1 Q/2) r P_1$</td>
</tr>
<tr>
<td>2</td>
<td>$T_2$</td>
<td>$X_2 Q/2$</td>
<td>$(X_2 Q/2) r P_2$</td>
</tr>
<tr>
<td>$i$</td>
<td>$T_i$</td>
<td>$X_i Q/2$</td>
<td>$(X_i Q/2) r P_i$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$n$</td>
<td>$T_n$</td>
<td>$X_n Q/2$</td>
<td>$(X_n Q/2) r P_n$</td>
</tr>
</tbody>
</table>

Therefore, the total holding cost per period (THCP) is

$$\text{THCP} = \frac{X_1 Q r P_1}{2} T_1 + \frac{X_2 Q r P_2}{2} T_2 + \cdots + \frac{X_n Q r P_n}{2} T_n.$$ 

On the other hand,

$$T_i = \frac{X_i Q}{D}, \quad i = 1, 2, \ldots, n.$$ 

Hence

$$\text{THCP} = \frac{X_1 Q r P_1 X_2 Q}{2 D} + \frac{X_2 Q r P_2 X_2 Q}{2 D} + \cdots + \frac{X_n Q r P_n X_n Q}{2 D},$$

$$\text{THCP} = \frac{r Q^2}{2D} \left( \sum_{i=1}^{n} X_i^2 P_i \right).$$

As annual holding cost (AHC) is

Annual holding cost

$$= \text{(total holding cost per period)} \times \text{(number of periods per year)},$$

$$\text{AHC} = \left( \frac{\text{THCP}}{D} \right) \frac{D}{Q}.$$ 

Therefore,

$$\text{AHC} = \frac{r Q^2}{2D} \left( \sum_{i=1}^{n} X_i^2 P_i \right).$$

2.3. Annual purchasing cost (APC)

By considering that the annual purchased quantity from the $i$th supplier is $X_i D$ and the price is equal to $P_i$, The APC is

$$\text{APC} = \sum_{i=1}^{n} X_i P_i D.$$ 

Therefore, the total cost of purchasing (TAPC) is

$$\text{TAPC} = \left( \sum_{i=1}^{n} A_i Y_i \right) D + \frac{r Q}{2} \left( \sum_{i=1}^{n} X_i^2 P_i \right) + \sum_{i=1}^{n} P_i X_i D.$$ 

As $Q$ is the optimum order quantity, it can be calculated by using the derivative of TAPC:

$$\frac{\partial (\text{TAPC})}{\partial Q} = 0 \Rightarrow Q = \sqrt[Q]{\frac{2D}{r} \left( \sum_{i=1}^{n} A_i Y_i \right) \left( \sum_{i=1}^{n} X_i^2 P_i \right)}.$$ 

By substituting for $Q$ in the TAPC, it becomes

$$\text{TAPC} = \sqrt{2D} \left( \sum_{i=1}^{n} A_i Y_i \right) \left( \sum_{i=1}^{n} X_i^2 P_i \right) + \sum_{i=1}^{n} P_i X_i D.$$ 

2.4. Constraints

The most important constraints of the problem are: supplier capacity, buyer's demand, and quality. These constraints are formulated as follows.
2.4.1. Capacity constraint

The vendor $i$ can provide up to $C_i$ units of the product over the year. Therefore, the purchased units from this vendor should be less than $C_i$ in each year. On the other hand,

$$(\text{Purchased units per year}) = (\text{Purchased units per period}) \times (\text{number of periods per year}).$$

Therefore,

$purchased units from \text{th supplier per year}$

$$= X_i Q \times 1/T = X_i Q \times D/Q = X_i D.$$

Hence,

$X_i D \leq C_i, \quad i = 1, 2, \ldots, n.$

2.4.2. Demand constraint

As demand is equal to $D$ over a year and it is assumed that $n$ vendors can meet the buyer’s demand, then,

$$\sum_{i=1}^{n} X_i D = D,$$

$$\sum_{i=1}^{n} X_i = 1.$$

2.4.3. Quality constraint

As $q_a$ is the buyer’s minimum acceptable perfect rate and $q_i$ is the perfect rate of the vendor $i$, and annual purchased volume is $X_i D$, the quality constraint can be shown as

$$\sum_{i=1}^{n} X_i D q_i \geq q_a q_a D,$$

$$\sum_{i=1}^{n} X_i q_i \geq q_a.$$

After constructing all constraints, it is necessary to be sure of the integer variables’ conditions, which are: if $Y_i$ is zero, $X_i$ is also zero and if $Y_i$ is 1, $X_i$ must be greater than zero. By considering that $X_i$ is less than one. The following constraints can satisfy these conditions:

$X_i \leq Y_i, \quad i = 1, 2, \ldots, n,$

$X_i \geq \varepsilon Y_i, \quad i = 1, 2, \ldots, n,$

where $\varepsilon$ is slightly greater than zero.

2.5. Final model

The final integer non-linear programming model is

$$\text{Min}(\text{TAPC}) = \sqrt{2D \sum_{i=1}^{n} A_i Y_i \left( \sum_{i=1}^{n} X_i^2 P_i \right)} + \sum_{i=1}^{n} P_i X_i D$$

s.t.

$$\sum_{i=1}^{n} X_i q_i \geq q_a,$$

$$X_i D \leq C_i, \quad i = 1, 2, \ldots, n,$$

$$X_i \leq Y_i, \quad i = 1, 2, \ldots, n,$$

$$X_i \geq \varepsilon Y_i, \quad i = 1, 2, \ldots, n,$$

$$\sum_{i=1}^{n} X_i = 1,$$

$$X_i \geq 0, \quad Y_i = 0, 1, \quad i = 1, 2, \ldots, n.$$

3. Model solution algorithm

General purpose software packages for solving non-linear programming, such as GINO [28] or Solver from Excel, require that all variables be continuous. This problem is a mixed integer non-linear programming. By branching the integer variables ($Y_i$ which is a binary variable), and substituting their values in the programming, the problem becomes a pure non-linear programming which can be solved by GINO or Solver. If there are $n$ suppliers, $2^n$ pure non-linear problems should be solved. Fortunately the combinations which can not satisfy the demand can be omitted, hence the following algorithm is proposed for the model solution:

1. Make a list of all combinations of $Y_i$s (at most $2^n$ times).
2. Omit the cases, which can not satisfy the demand constraint.
3. Substitute the values of $Y_i$s in the integer programming to change it to pure non-linear programming (PNP). If the set $\{S\}$ is defined as the
set of $Y_i$s for which their values are equal to one, the pure nonlinear programming becomes

$$\text{Min} (\text{TAPC}) = \sqrt{2Dr \left( \sum_{i \in S} A_i \right) \left( \sum_{i = 1}^{n} P_i X_i^2 D \right)} + \sum_{i = 1}^{n} P_i X_i D$$

s.t.

$$\varepsilon \leq X_i \leq \frac{C_i}{D_i}, \quad i \in S,$$

$$X_i = 0, \quad i \notin S,$$

$$\sum_{i = 1}^{n} X_i = 1,$$

$$\sum_{i = 1}^{n} X_i q_i \geq q_i, \quad i = 0, 1, 2, \ldots, n.$$

4. Use GINO or Solver to solve the PNPs and find the best solution for each case. In general, the answer which is found by the package, can be either the local or global optimum. The user should approve that it is a global one. In order to verify that the optimum solution is a global one, it is necessary to show that the objective function is convex [28,29]. The convexity of the model is discussed in the appendix.

5. Choose the minimum TAPC from all the feasible cases as the best answer. This algorithm is shown in Fig. 2.

4. Numerical example

Assume that the management of a company would like to find the minimum cost of suppliers. Three suppliers are to be evaluated, their information is presented in Table 1.

4.1. Situation 1

The objective is to find the best choice of suppliers if the demand is 1000, $r = 0.2$ and the minimum accepted perfect rate is 0.95.

For simplicity, the solution will be described according to the algorithm's steps.

**Step 1:** As three suppliers should be evaluated, there are $8 \ (= 2^3)$ possibilities of integer programming, which are listed in the Table 2.

**Step 2:** The capacity of selected suppliers in each case is shown in Table 3.

### Table 1
<table>
<thead>
<tr>
<th>Suppliers' information</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Price</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Sup. 1</td>
</tr>
<tr>
<td>Sup. 2</td>
</tr>
<tr>
<td>Sup. 3</td>
</tr>
</tbody>
</table>

### Table 2
<table>
<thead>
<tr>
<th>Eight possibilities of integer variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cases</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
</tbody>
</table>

Fig. 2. Flowchart of the model solution.
Table 3
Feasible and unfeasible cases (F = feasible, U = unfeasible)

<table>
<thead>
<tr>
<th>Cases</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
<th>( Y_3 )</th>
<th>Capacities</th>
<th>Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1800</td>
<td>F</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1300</td>
<td>F</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1100</td>
<td>F</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1200</td>
<td>F</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>500</td>
<td>U</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>700</td>
<td>U</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>600</td>
<td>U</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>U</td>
</tr>
</tbody>
</table>

Table 4
Solver answer report for case 1

Microsoft Excel 5.0c
Answer Report
Worksheet: [TOTCOST1.XLS]
Sheet1 (5)
Report Created: 29/8/96 12:04

Target Cell (Min)

<table>
<thead>
<tr>
<th>Name</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target cell</td>
<td>17621.31</td>
</tr>
</tbody>
</table>

Adjustable Cells

<table>
<thead>
<tr>
<th>Name</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order Qua. SUP1</td>
<td>0.15</td>
</tr>
<tr>
<td>Order Qua. SUP2</td>
<td>0.70</td>
</tr>
<tr>
<td>Order Qua. SUP3</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Step 3: In the first case, of how many should be purchased from which of the three suppliers, the PNP model can be shown as

\[
\text{Min}(TACP) = \sqrt{8400(9X_1^2 + 16X_2^2 + 32X_3^2)} + 1000(9X_1 + 16X_2 + 32X_3)
\]

s.t.

\[
0.98X_1 + 0.95X_2 + 0.92X_3 \geq 0.95, \\
0.01 \leq X_1 \leq 0.6, \\
0.01 \leq X_2 \leq 0.7, \\
0.01 \leq X_3 \leq 0.5, \\
X_1 + X_2 + X_3 = 1, \\
X_i \geq 0, \quad i = 1, 2, 3.
\]

Table 5
Optimum solution for the satisfied demand cases

<table>
<thead>
<tr>
<th>Cases</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( X_3 )</th>
<th>( Q )</th>
<th>TC</th>
<th>TQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>0.70</td>
<td>0.15</td>
<td>154</td>
<td>17621</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>0.70</td>
<td>0.00</td>
<td>—</td>
<td>Unfeasible</td>
<td>0.94</td>
</tr>
<tr>
<td>3</td>
<td>0.50</td>
<td>0.00</td>
<td>0.50</td>
<td>128</td>
<td>20764</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>0.70</td>
<td>0.30</td>
<td>105</td>
<td>21026</td>
<td>0.96</td>
</tr>
</tbody>
</table>

In order to solve this PNP, the Solver from Microsoft Excel is applied. The optimum order allocations assigned to each supplier, as the Solver answer report shows in Table 4, are, respectively, in proportion 0.15, 0.70 and 0.15. This means that the optimum order quantity is 154 and the order quantities which should be purchased from suppliers, respectively, are 23, 108 and 23.

Step 4: The optimum answers for all the cases are shown in Table 5. It should be noticed that case two is unfeasible for its low quality.

As Table 5 shows, case 1 has the minimum cost and therefore is the optimum solution.

4.2. Situation 2

Assume, instead, that the minimum acceptable perfect rate reduces to 0.92, the optimum order allocations to the three suppliers respectively are 0.60, 0.40, 0 and the total cost of purchasing is 11973. The Solver solutions are shown in Table 6.

4.3. Situation 3

If the demand and quality, respectively, becomes 400 and 0.96, the best order allocations are 0, 0.67, 0.33 and the total cost of purchasing is 8676. The Solver solutions are shown in Table 7.

4.4. Situation 4

In the condition of \( D = 400 \) and \( q_a = 0.92 \), the Solver solutions which are shown in Table 8, reveal that all demand should be bought from supplier one to give the minimum total cost.

The answers to these four situations are compared in Table 9.
In the conditions of \( D = 1000 \) and \( q_a = 0.95 \), 15\% of demand is purchased from supplier one whereas in the case of \( D = 1000 \) and \( q_a = 0.92 \), 50\% of demand is assigned to this supplier. Comparison of the cost and quality of these two cases reveals that in the first case the buyer prefers to pay more for better quality.

In the case of \( D = 400 \) and \( q_a = 0.96 \), although all the suppliers can satisfy the buyer’s demand, they cannot meet the quality requirement. Therefore, the buyer should buy from suppliers two and three. Under a policy of one supplier only the buyer should buy from supplier three, which can satisfy both the demand and quality needs, but its cost is greater than a two supplier policy.

In the case of \( D = 400 \) and \( q_a = 0.92 \), all suppliers can satisfy the buyer requirements of quality and demand. Therefore, the problem reverts to the simple traditional EOQ model to analyse the single supplier. As supplier one gives the minimum total cost, it is selected by the model, which means that for the single supplier the general model gives the same results as the traditional EOQ model.

### 5. A multiple objective programming model

In the single objective model of Section 2, for supplier selection two important criteria are considered: the first is cost, as the objective function, and the second is quality, as the constraint in the programming.

In practice there may be several criteria for supplier selection such as on-time delivery, after sale service, response to change, etc. In order to take these criteria into account, they should be considered as constraints in the programming. In all single objective models it is implicitly assumed that these criteria, which are considered as constraints, have the same priority. This rarely happens in practice because decision makers may wish to prioritize their criteria differently. To address this situation, a multiple objective programming model has been developed to help the decision makers reflect corporate strategies in their supplier selection.

The idea of multiple objective programming is to minimize the deviations from each goal, in order of
priority. Higher priority goals are satisfied at the expense of lower priority goals. The multiple objective programming model of Section 2, which considers the cost and quality as the criteria, becomes

\[
\text{Min } Z = w_1 \left[ \sqrt{2Dr} \left( \sum_{i=1}^{n} A_i Y_i \right) \left( \sum_{i=1}^{n} X_i^2 P_i \right) + \sum_{i=1}^{n} P_i X_i D - c^* \right] + w_2 \left[ q^* - \sum_{i=1}^{n} X_i q_i \right]
\]

s.t.
\[
\sum_{i=1}^{n} X_i q_i \geq q_a,
\]
\[
X_i D \leq C_i, \quad i = 1, 2, \ldots, n,
\]
\[
X_i \leq Y_i, \quad i = 1, 2, \ldots, n,
\]
\[
X_i \geq c Y_i, \quad i = 1, 2, \ldots, n,
\]
\[
\sum_{i=1}^{n} X_i = 1
\]
\[
X_i \geq 0, \quad Y_i = 0, 1, \quad i = 1, 2, \ldots, n.
\]

In which \(c^*, q^*, w_2\) and \(w_2\), respectively, are the optimum cost, quality, and the weightings for cost and quality.

The solution to this NIP is similar to the NIP of Section 2, because the objective function still is a convex function, and the constraints are linear. Therefore, the same procedure can be used to solve the model.

It should be noticed that \(c^*\) and \(q^*\) are the optimum values. If they are less than their optimum, the objective function becomes an absolute value and its solution will be different.

6. Summary and conclusions

Supplier selection is one of the most important activities of purchasing managers in which cost, quality, delivery, etc., should be considered in selecting the best suppliers. Shortage of suppliers' capacity makes the problem difficult, and considering the total cost of purchasing makes it more complicated. This paper has described a non-linear integer programming model which has been developed to help managers in this decision making. In order to solve the non-linear integer programming, it is necessary to solve \(2^n\) pure non-linear programs. Although the model should be run \(2^n\) times for \(n\) suppliers, the model solution should not take too long because in most practical cases there are usually a maximum of 12 vendors [21] and also because some cases are omitted, as they cannot satisfy the demand constraint.

The model has a number of advantages:

1. It can consider multiple criteria such as cost, quality etc. in supplier selection problems.
2. The total cost of purchasing rather than just price, can be included in the decision making process. Total cost contains transportation, inspection, ordering and storage costs.
3. The model can calculate the economic order quantities (EOQ) for both single and multiple sourcing with and without constraints.
4. The model enables the management to reflect corporate strategies in the purchasing activities.
5. A schedule for deliveries can be provided, which tells the buyer when and how much should be purchased from each supplier.
6. As the model can be solved using Solver from Microsoft Excel, it is user-friendly and easy to apply by the purchasing management.

Appendix. Model convexity

To approve that the function \(f\) is convex, the Hessian matrix of \(f\) must be positive semi-definite. Every matrix is positive semi-definite if

(a) all diagonal elements are non-negative,
(b) all the principal determinants are non-negative.

Full discussion of this subject is presented in Ravindran et al. [30]. The objective function of the programming is

\[
\text{TAPC} = \sqrt{2D \sum_{i=1}^{n} (A_i) \left( \sum_{i=1}^{n} X_i^2 P_i \right) + \sum_{i=1}^{n} P_i X_i D}.
\]
As the sum of two convex functions is convex and also as the second term of the objective function is linear, it is necessary to approve that the first term determinant of the above matrix is equal to the determinant of the following matrix, multiplied by 

\[ \begin{vmatrix} P_1 X_1 P_2 X_2 \ldots P_n X_n / T^{(3/2)} \\ (T - P_1 X_1^2) / X_1 & - P_2 X_2 \\ - P_1 X_1 & (T - P_2 X_2^2) / X_2 \\ - P_1 X_1 & - P_2 X_2 \\ \vdots & \vdots \\ - P_1 X_1 & - P_2 X_2 \end{vmatrix} \cdot (-1) \text{ and then added to each row, the determinant is equal to the determinant of the following matrix:} 

\[ \begin{vmatrix} T/X_1 & 0 & 0 & \ldots & -T/X_n \\ 0 & T/X_2 & 0 & \ldots & -T/X_n \\ 0 & 0 & T/X_3 & \ldots & -T/X_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ - P_1 X_1 & - P_2 X_2 & - P_3 X_3 & \ldots & (T - P_n X_n^2) / X_n \end{vmatrix} \]

If each column of the above matrix are multiplied by \( X_i / X_n \), then the determinant of the above matrix becomes equal to the determinant of the following matrix multiplied by \( P_1 P_2 \ldots P_n X_n^n T^{(3/2)} \):

\[ \begin{vmatrix} T/X_1 & 0 & 0 & \ldots & -T/X_n \\ 0 & T/X_2 & 0 & \ldots & -T/X_n \\ 0 & 0 & T/X_3 & \ldots & -T/X_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ - P_1 X_1^2 / X_n & - P_1 X_1^2 / X_n & - P_3 X_3^2 / X_n & \ldots & (T - P_n X_n^2) / X_n \end{vmatrix} \]

By dividing all the rows by the corresponding \( P_j X_j \) (for \( j = 1 \) to \( n \)) and using the properties of the determinant of square matrices which are stated at the end of this appendix it can be seen that the determinant of the above matrix is equal to the determinant of the following matrix multiplied by \( P_1 P_2 \ldots P_n X_n^n / T^{(3/2)} \):
\[
\begin{vmatrix}
T/X_n & 0 & 0 & \ldots & 0 \\
0 & T/X_n & 0 & \ldots & 0 \\
0 & 0 & T/X_n & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\end{vmatrix}
= -P_1X_1^2/X_n - P_1X_2^3/X_n - P_3X_3^2/X_n \ldots 0
\]

The determinant of the above matrix and its \( k \)th order principal matrix are non-negative, thus the \( f \) (objective) function is convex.

### A.1. Properties of the determinant

The most important properties of the determinant of square matrices are as follows [31]:

1. If there exists any row (or any column) all of whose elements are zero, the determinant is zero.
2. If the elements in any two rows (or any two columns) are the same, the determinant is zero.
3. If the elements of any row (or any column) are multiplied by a scalar, then the determinant of the new matrix is equal to the determinant of the old matrix multiplied by the scalar.
4. Any scalar multiple of any row (or column) can be added to any other row (or column) without changing the value of the determinant.

### References


